Interaction Surplus Framework: Papers A-E

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Uniqueness and Properties of Interaction Surplus Functionals on Block-Decomposed Inner Product Spaces

Paper A (Final)

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Abstract. Let $V = \mathbb{R}^{NM}$ with $N, M \geq 2$, orthogonally decomposed into N subspaces of dimension M. We prove that the surplus functional $F(\vec{x}, \vec{y}) = f(u(\vec{x}, \vec{y}))$ with $f(u) = \ln(1 + (N-1)u)$ is uniquely characterized by four axioms: geometric dependence (S1), zero at zero (S2), affine effective count (S3), and normalization (S4). The uniqueness proof requires five lines of algebra. Strict monotonicity and strict concavity are derived consequences, not axioms. We prove decomposition, sharp Lipschitz stability, extremality, and diagonal surplus theorems. The scalar axioms S2–S4 are independent relative to S1 via separate witnesses, and monotonicity/concavity are derived rather than assumed.

Contribution. This paper gives an axiomatic characterization of surplus functionals $F : S(V) \times S(V) \to \mathbb{R}_{\geq 0}$ on the unit sphere of a block-decomposed Euclidean space $V = \mathbb{R}^{NM}$ that satisfy

the reduction axiom S1 together with the scalar axioms S2–S4. We prove that the scalar profile is uniquely forced to be $f(u) = \ln(1 + (N-1)u)$, and from this derive strict monotonicity, positivity, strict concavity, subadditivity on the interaction parameter, two-source decomposition, sharp Lipschitz stability, extremality, and diagonal-surplus positivity. We also prove that, relative to S1, the scalar axioms S2, S3, and S4 are independent. The paper is purely formal and makes no empirical or metaphysical claims.

Introduction

This paper studies scalar interaction functionals on the unit sphere of a finite-dimensional real inner-product space when the interaction variable is the angle-derived quantity

$$u(\vec{x}, \vec{y}) = 1 - (\vec{x} \cdot \vec{y})^2.$$

The resulting class is natural whenever state comparison is orientation-based rather than sign-sensitive, and when one wishes to measure interaction strength by a bounded scalar on $[0, 1]$. The additional block decomposition $V = H_1 \oplus \cdots \oplus H_N$ is not needed to state the scalar characterization theorem itself, but becomes essential for the later structural results, especially the crossing/divergence decomposition and the diagonal-surplus theorem.

The mathematical interest of the paper is modest and precise. It is not the discovery of a new logarithmic quantity in general. Rather, it is the exact axiomatization of one specific log-surplus profile on the sphere together with a clean package of structural consequences. The main contributions are therefore: first, a short characterization theorem showing that S1–S4 force $f(u) = \ln(1+(N-1)u)$; second, a proof that, relative to the reduction axiom S1, the scalar axioms S2–S4 are independent; and third, the derivation of structural consequences that become available once the scalar law is fixed, including the two-source decomposition theorem and the sharp Lipschitz bound.

The problem is adjacent to several established literatures but does not coincide with any of them. Classical axiomatic work on information measures characterizes logarithmic forms on probability simplices and discrete distributions; see, for example, Shannon, Khinchin, and Rényi. By contrast, the present paper works on the unit sphere and characterizes a scalar profile in an angle-derived variable. The geometric side is closer to the literature on bivectors, principal angles, and subspace geometry, where $\|\vec{x} \wedge \vec{y}\|$ and related angle quantities are basic invariants. The block-structured side is adjacent to direct-sum and frame-based settings in which interaction across components or subspaces matters. The novelty claimed here is therefore narrow: not a new theory of entropy, not a general theory of information geometry, and not a theory of subspaces, but an axiomatic characterization of a particular surplus law together with its block-structured consequences.

Related and neighboring work

Three neighboring strands are especially relevant. First, the logarithmic form places the paper near axiomatic characterizations of information measures, but the domain and axioms are different: the current characterization is on $[0, 1]$ through an angle-derived parameter, not on probability vectors. Second, the use of $u(\vec{x}, \vec{y}) = \|\vec{x} \wedge \vec{y}\|^2$ places the work near exterior-algebra and principal-angle geometry, where wedge norms encode orientation-dependent separation. Third, the decomposition $V = H_1 \oplus \cdots \oplus H_N$ places the later theorems near block-structured linear algebra and frame-theoretic settings in which direct-sum organization matters. The paper does not attempt to subsume those literatures; it isolates one function class that sits at their intersection.

0. Standing Assumptions and Notation

- All vector spaces are real finite-dimensional inner-product spaces.
- $V = \mathbb{R}^{NM}$ with $N, M \geq 2$, orthonormal basis $\{e_{ij}\}$ where $1 \leq i \leq N$, $1 \leq j \leq M$.
- $H_i = \text{span}\{e_{i1}, \ldots, e_{iM}\}$; blocks are mutually orthogonal.
- $S(V) = \{\vec{x} \in V : \|\vec{x}\| = 1\}$.
- $u(\vec{x}, \vec{y}) = 1 - (\vec{x} \cdot \vec{y})^2 = \|\vec{x} \wedge \vec{y}\|^2 \in [0, 1]$.
- Surplus functionals take values in $\mathbb{R}_{\geq 0}$.
- All logarithms are natural logarithms.

Architectural remark. The characterization theorem in §2 is a sphere-level statement: once S1 reduces the problem to a scalar profile on $u \in [0, 1]$, the proof of uniqueness uses only S2–S4. The block decomposition is used essentially only in the later structural theorems, where crossing and within-block divergence are separated.

1. Axioms

A *surplus functional* is a function $F : S(V) \times S(V) \to \mathbb{R}_{\geq 0}$.

S1 *(Geometric dependence).* There exists $f : [0, 1] \to \mathbb{R}_{\geq 0}$ such that $F(\vec{x}, \vec{y}) = f(u(\vec{x}, \vec{y}))$.

S2 *(Zero at zero).* $f(0) = 0$.

S3 *(Affine effective count).* The function $g(u) = e^{f(u)}$ is affine: $g(u) = au + b$ for some $a, b \in \mathbb{R}$.

S4 *(Normalization).* $f(1) = \ln N$.

Remark 1.1. S2 states that surplus vanishes when the interaction parameter $u = 0$. S3 is the *design axiom*: it asserts that the effective number of accessible states grows linearly in the interaction parameter. Combined with S2 and S4, it forces the unique scalar profile. S3 is not derived from more primitive assumptions in this paper; it should be read as the axiom that selects the function class under study.

Remark 1.2. The stronger statement "zero iff parallel" is derived in Theorem 4.3(a) after establishing monotonicity: $F(\vec{x}, \vec{y}) = 0 \iff f(u) = 0 \iff u = 0 \iff \vec{x} = \pm\vec{y}$. S2 assumes only vanishing at $u = 0$; the converse is derived from strict monotonicity.

Why These Four Axioms?

Why S1 is natural. The quantity $u(\vec{x}, \vec{y}) = 1 - (\vec{x} \cdot \vec{y})^2$ is the canonical orientation-based interaction parameter on the unit sphere: it is invariant under sign changes, bounded in $[0, 1]$, and determined entirely by the relative angle between the states. Requiring F to depend only on u isolates the interaction geometry from coordinate choices inside each block.

Why S2 is natural. When $u = 0$, the states are parallel or antiparallel and therefore introduce no angular difference. The zero-baseline condition $f(0) = 0$ encodes the minimal requirement that zero geometric difference yields zero surplus.

Why S4 is natural. The value $u = 1$ is the maximal point of the interaction parameter, attained exactly at orthogonality. The normalization $f(1) = \ln N$ fixes the scale by the number of blocks and identifies maximal surplus with maximal geometric separation.

Why S3 is natural. Writing $g(u) = e^{f(u)}$ interprets surplus on an effective-count scale. Once the endpoint constraints are fixed by S2 and S4, affine growth is the unique parameter-free interpolation

$g(0) = 1$, $g(1) = N$. Higher-order or nonlinear alternatives introduce extra curvature parameters or additional structural assumptions. S3 is therefore best understood as a deliberately chosen axiom-schema selecting the simplest parameter-free effective-count law compatible with the endpoint data, not as a theorem forced by S1, S2, and S4 alone.

2. The Characterization Theorem

Theorem 2.1 *(Uniqueness).* Under S1–S4, the unique scalar profile compatible with the surplus functional is:

$$\boxed{f(u) = \ln(1 + (N-1)\,u)}$$

Proof. By S3, $g(u) = au + b$ for some $a, b \in \mathbb{R}$. By S2, $f(0) = 0$, hence $g(0) = e^{f(0)} = 1$. Therefore $b = 1$. By S4, $f(1) = \ln N$, hence $g(1) = e^{f(1)} = N$. Therefore $a + b = N$, so $a = N - 1$. Substituting these values gives $g(u) = 1 + (N-1)u$. Since $g(u) = e^{f(u)}$, we obtain $f(u) = \ln(1 + (N-1)u)$. □

This proof uses only the four stated axioms, invokes no external results, and introduces no hidden assumptions. In particular, once S1 has reduced the problem to a scalar profile on $[0, 1]$, the uniqueness argument does not use the block decomposition.

3. Derived Properties

The following are *consequences* of the characterized f, not additional axioms.

Theorem 3.1 *(Strict Monotonicity).* f is strictly increasing on $[0, 1]$.

Proof. $f'(u) = \frac{N-1}{1+(N-1)u} > 0$ for $N \geq 2$, $u \in [0, 1]$. Therefore f is strictly increasing. □

Theorem 3.2 *(Positivity).* $f(u) > 0$ for all $u > 0$.

Proof. By Theorem 3.1, f is strictly increasing. Thus for $u > 0$: $f(u) > f(0) = 0$. Hence $f(u) > 0$. □

Theorem 3.3 *(Strict Concavity).* f is strictly concave on $[0, 1]$.

Proof. $f''(u) = -\frac{(N-1)^2}{(1+(N-1)u)^2} < 0$. Therefore f is strictly concave. □

Theorem 3.4 *(Subadditivity on parameter).* For $a, b \geq 0$ with $a + b \leq 1$: $f(a + b) \leq f(a) + f(b)$.

Proof. By Theorem 3.3, f is strictly concave, hence f' is strictly decreasing. Therefore:

$$f(a + b) - f(a) = \int_a^{a+b} f'(t)\,dt \leq \int_0^b f'(t)\,dt = f(b)$$

Thus $f(a + b) \leq f(a) + f(b)$. □

Remark 3.5. Subadditivity on the parameter u does *not* imply a triangle inequality for geometric triples. The parameterization $u = \sin^2\theta$ is not additive in angle. See §5 for a counterexample.

4. Structural Theorems

4.1 Decomposition

Theorem 4.1 *(Two-Source Decomposition).* Let $\vec{x} \in H_i$ be a pure block vector. For any unit $\vec{y}$, decompose $\vec{y} = \vec{y}^{\parallel} + \vec{y}^{\perp}$ with $\vec{y}^{\parallel} \in H_i$, $\vec{y}^{\perp} \perp H_i$. Let $\alpha = \|\vec{y}^{\parallel}\|$, $\beta = \|\vec{y}^{\perp}\|$, and $\gamma = \angle(\vec{x}, \vec{y}^{\parallel})$. Then:

$$u(\vec{x}, \vec{y}) = \underbrace{\beta^2}_{u_{\text{cross}}} + \underbrace{\alpha^2 \sin^2 \gamma}_{u_{\text{div}}}$$

and:

$$\max(f(u_{\text{cross}}), f(u_{\text{div}})) \leq f(u) \leq f(u_{\text{cross}}) + f(u_{\text{div}})$$

Proof. Since $\vec{x} \cdot \vec{y} = \vec{x} \cdot \vec{y}^{\parallel} = \alpha \cos \gamma$, we have:

$$u = 1 - \alpha^2 \cos^2 \gamma = (1 - \alpha^2) + \alpha^2 \sin^2 \gamma = \beta^2 + \alpha^2 \sin^2 \gamma$$

Thus $u = u_{\text{cross}} + u_{\text{div}}$. By Theorem 3.1 (monotonicity) and Theorem 3.4 (subadditivity), the bounds follow. □

Remark 4.1.1. The point of Theorem 4.1 is not merely the algebraic identity for u, which is elementary once the decomposition is written down. The structural content is that the scalar surplus law interacts cleanly with the split into crossing and divergence terms, producing monotonicity and subadditivity bounds from the characterization theorem.

4.2 Stability

Theorem 4.2 *(Sharp Lipschitz Stability).* For all $u_1, u_2 \in [0, 1]$,

$$|f(u_1) - f(u_2)| \leq (N-1)|u_1 - u_2|.$$

Moreover, the constant $N-1$ is the smallest global Lipschitz constant of f on $[0, 1]$.

Proof. Since

$$f'(u) = \frac{N-1}{1 + (N-1)u},$$

we have $0 < f'(u) \leq N - 1$ for all $u \in [0, 1]$. The mean value theorem therefore gives

$$|f(u_1) - f(u_2)| \leq \sup_{u \in [0,1]} |f'(u)| \, |u_1 - u_2| = (N-1)|u_1 - u_2|.$$

To prove sharpness, let $t > 0$. Then

$$\frac{|f(t) - f(0)|}{|t - 0|} = \frac{\ln(1 + (N-1)t)}{t}.$$

As $t \downarrow 0$, this quotient converges to $f'(0) = N - 1$. Hence any global Lipschitz constant $L < N - 1$ fails for sufficiently small t. Therefore $N - 1$ is optimal. □

4.3 Extremality

Theorem 4.3. (a) $F(\vec{x}, \vec{y}) = 0$ iff $\vec{x} = \pm\vec{y}$. (b) $F(\vec{x}, \vec{y}) = \ln N$ iff $\vec{x} \perp \vec{y}$. (c) For k mutually orthogonal unit vectors from k distinct blocks, total pairwise surplus is $\binom{k}{2} \ln N$.

Proof.

- (a) $F(\vec{x}, \vec{y}) = 0 \iff f(u) = 0$. By Theorem 3.1, f is strictly increasing and $f(0) = 0$, so $f(u) = 0 \iff u = 0$. But $u = 0 \iff (\vec{x} \cdot \vec{y})^2 = 1 \iff \vec{x} = \pm\vec{y}$.
- (b) $F(\vec{x}, \vec{y}) = \ln N = f(1)$. By strict monotonicity, this forces $u = 1$. Since $u = 1 - (\vec{x} \cdot \vec{y})^2$, we get $(\vec{x} \cdot \vec{y})^2 = 0 \iff \vec{x} \perp \vec{y}$.
- (c) Pairwise orthogonality gives $u = 1$ for each pair, so each pair contributes $\ln N$, totaling $\binom{k}{2} \ln N$. □

4.4 Diagonal Surplus

Theorem 4.4. Let $\hat{e}_X = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} e_{i1}$ (diagonal across first rooms). For any pure block vector $\vec{x} \in H_i$:

$$F(\vec{x}, \hat{e}_X) = \ln\left(1 + (N-1)\left(1 - \frac{(\vec{x} \cdot e_{i1})^2}{N}\right)\right) > 0$$

Proof. Since $\vec{x} \in H_i$, we have

$$\vec{x} \cdot \hat{e}_X = \frac{1}{\sqrt{N}}(\vec{x} \cdot e_{i1}).$$

Therefore

$$u(\vec{x}, \hat{e}_X) = 1 - \frac{(\vec{x} \cdot e_{i1})^2}{N}.$$

Because $\|\vec{x}\| = 1$, we have $(\vec{x} \cdot e_{i1})^2 \leq 1$, hence

$$u(\vec{x}, \hat{e}_X) \geq 1 - \frac{1}{N} > 0$$

for $N \geq 2$. Substituting this value of u into Theorem 2.1 gives the stated formula, and positivity follows from Theorem 3.2. □

5. Why Not a Triangle Inequality?

A previous version included an axiom requiring $f(u_{xz}) \leq f(u_{xy}) + f(u_{yz})$ for geometric triples. This is **incompatible** with the characterized f.

Counterexample. Three coplanar unit vectors with $\theta_{xy} = \theta_{yz} = 0.1$, $\theta_{xz} = 0.2$, $N = 12$:

Quantity	Value
$u_{xy} = u_{yz}$	$\sin^2(0.1) \approx 0.00998$
u_{xz}	$\sin^2(0.2) \approx 0.03947$
$f(u_{xz})$	≈ 0.361
$f(u_{xy}) + f(u_{yz})$	≈ 0.209

Since $0.361 > 0.209$, the triangle inequality fails. The reason: $u = \sin^2\theta$ is not additive in θ; for small angles, $(\theta_{xy} + \theta_{yz})^2 > \theta_{xy}^2 + \theta_{yz}^2$.

6. Axiom Structure, Independence, and Alternatives

6.1 Relative Independence

Definition 6.1. Fix $N, M \geq 2$. A scalar axiom among $\{\mathrm{S2}, \mathrm{S3}, \mathrm{S4}\}$ is said to be *independent relative to S1* if there exists a surplus functional $F(\vec{x}, \vec{y}) = f(u(\vec{x}, \vec{y}))$ satisfying S1 and the other two scalar axioms but failing the chosen axiom. This is the relevant notion here because S1 is the structural reduction hypothesis that makes the scalar profile f available.

Proposition 6.2 *(S2 is independent relative to S1).* Define

$$f_{\mathrm{S2}}(u) = \ln(2 + (N-2)u).$$

Then $F_{\mathrm{S2}}(\vec{x}, \vec{y}) = f_{\mathrm{S2}}(u(\vec{x}, \vec{y}))$ satisfies S1, S3, and S4, but fails S2.

Proof. S1 holds by construction. Since

$$e^{f_{\mathrm{S2}}(u)} = 2 + (N-2)u,$$

the effective-count map is affine, so S3 holds. Also $f_{\mathrm{S2}}(1) = \ln N$, so S4 holds. But $f_{\mathrm{S2}}(0) = \ln 2 \neq 0$, so S2 fails. □

Proposition 6.3 *(S3 is independent relative to S1).* Define

$$f_{\mathrm{S3}}(u) = u \ln N.$$

Then $F_{\mathrm{S3}}(\vec{x}, \vec{y}) = f_{\mathrm{S3}}(u(\vec{x}, \vec{y}))$ satisfies S1, S2, and S4, but fails S3.

Proof. S1 holds by construction. Since $f_{\mathrm{S3}}(0) = 0$, S2 holds. Since $f_{\mathrm{S3}}(1) = \ln N$, S4 holds. But

$$e^{f_{\mathrm{S3}}(u)} = N^u,$$

which is not affine on $[0, 1]$, so S3 fails. □

Proposition 6.4 *(S4 is independent relative to S1).* Let $c > 0$ with $c \neq N - 1$, and define

$$f_{\mathrm{S4}}(u) = \ln(1 + cu).$$

Then $F_{\mathrm{S4}}(\vec{x},\vec{y}) = f_{\mathrm{S4}}(u(\vec{x},\vec{y}))$ satisfies S1, S2, and S3, but fails S4.

Proof. S1 holds by construction. Since $f_{\mathrm{S4}}(0) = \ln 1 = 0$, S2 holds. Also

$$e^{f_{\mathrm{S4}}(u)} = 1 + cu,$$

so S3 holds. But $f_{\mathrm{S4}}(1) = \ln(1+c) \neq \ln N$ because $c \neq N-1$, so S4 fails. □

Proposition 6.5 *(S1 is structurally indispensable for scalar characterization).* The axiom S1 is not an independence claim of the same type as S2–S4. Instead, it is the hypothesis that reduces the surplus functional to a scalar profile in the parameter u. In particular, the function

$$F(\vec{x},\vec{y}) = \|\vec{x}-\vec{y}\|^2$$

is not representable as $F(\vec{x},\vec{y}) = h(u(\vec{x},\vec{y}))$ for any scalar function $h : [0,1] \to \mathbb{R}$.

Proof. Let

$$\vec{x} = e_{11}, \qquad \vec{y} = \frac{e_{11}+e_{12}}{\sqrt{2}}, \qquad \vec{z} = \frac{-e_{11}+e_{12}}{\sqrt{2}}.$$

Then

$$\vec{x}\cdot\vec{y} = \frac{1}{\sqrt{2}}, \qquad \vec{x}\cdot\vec{z} = -\frac{1}{\sqrt{2}},$$

so

$$u(\vec{x},\vec{y}) = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} = 1 - \left(-\frac{1}{\sqrt{2}}\right)^2 = u(\vec{x},\vec{z}).$$

However,

$$F(\vec{x},\vec{y}) = \left\| e_{11} - \frac{e_{11}+e_{12}}{\sqrt{2}} \right\|^2 = 2 - \sqrt{2},$$

whereas

$$F(\vec{x},\vec{z}) = \left\| e_{11} - \frac{-e_{11}+e_{12}}{\sqrt{2}} \right\|^2 = 2 + \sqrt{2}.$$

Thus two pairs with the same value of u have different values of F, so no scalar profile $h(u)$ can represent this functional. □

Corollary 6.6 *(Independence plus derivability).* The scalar axioms $\{\mathrm{S2},\mathrm{S3},\mathrm{S4}\}$ are independent relative to S1, and no additional monotonicity or concavity axiom is needed.

Proof. Propositions 6.2–6.4 give one witness for each scalar axiom. Theorems 3.1 and 3.3 show that monotonicity and concavity are consequences of S2–S4 rather than additional assumptions. □

6.2 Alternative Effective-Count Laws and What They Lose

Proposition 6.7. Nearby alternatives to the characterized law lose at least one of the target properties of the surplus profile.

Proof.

- **Linear surplus:** If $f(u) = u \ln N$, then $f''(u) = 0$ on $[0, 1]$, so strict concavity fails.
- **Quadratic effective count:** If $g(u) = 1 + au + bu^2$ with $g(0) = 1$ and $g(1) = N$, then $a + b = N - 1$. One free parameter remains, so the endpoint constraints do not determine the law uniquely.
- **Exponential effective count:** If $g(u) = e^{cu}$ and $g(1) = N$, then $c = \ln N$, so $f(u) = u \ln N$. This again loses strict concavity.
- **Power-law effective count:** If $g_k(u) = 1 + (N-1)u^k$ with $k \neq 1$, then the exponent k is not fixed by the endpoint constraints. Moreover, for $0 < k < 1$,

$$f_k'(u) = \frac{(N-1)ku^{k-1}}{1 + (N-1)u^k}$$

is unbounded as $u \downarrow 0$, so global Lipschitz stability on $[0, 1]$ fails. For $k > 1$, we have $f_k'(0) = 0$, so the characterized origin slope $N - 1$ is lost.

In every case, at least one of strict concavity, parameter-freeness, or sharp linear response at the origin is lost. □

6.3 Why the Logarithm?

The logarithm is not an additional modeling choice once S3 is expressed through the effective-count map $g(u) = e^{f(u)}$. By definition, $f = \ln g$. What remains to justify is the use of the effective-count scale itself. The logarithm converts multiplicative effective counts into additive information increments, aligning the surplus with standard information-theoretic accounting. In addition, when g is affine, the logarithm yields a strictly increasing and strictly concave profile, whereas Proposition 6.7 shows that nearby alternatives either lose strict concavity or introduce extra free parameters. Thus the logarithm is the information scale induced by the effective-count representation, not an independent postulate beyond S3. No stronger uniqueness claim about arbitrary continuous monotone transforms is intended here.

7. Summary

Result	Statement	Proof
Theorem 2.1	$f(u) = \ln(1 + (N-1)u)$ uniquely determined under S1–S4	5 lines of algebra
Theorem 3.1	f strictly monotone	Derivative $f' > 0$
Theorem 3.3	f strictly concave	Second derivative $f'' < 0$
Theorem 4.1	Decomposition into crossing + divergence	Direct computation
Theorem 4.2	Sharp Lipschitz constant $N - 1$	Mean value theorem + optimality limit

Result	Statement	Proof
Theorem 4.3	Extrema at parallel (0) and orthogonal ($\ln N$)	Strict monotonicity + S4
Theorem 4.4	Diagonal vector has positive surplus with all pure states	Geometry
Propositions 6.2–6.4	Separate witnesses show S2, S3, and S4 are each independent relative to S1	Direct verification
Proposition 6.5	S1 is structurally necessary for scalar reduction to u	Counterexample with equal u but unequal surplus
Proposition 6.7	Nearby alternatives lose concavity, parameter-freeness, or sharp stability	Direct computation

8. What This Paper Proves

Proved:

- Under the reduction axiom S1 and scalar axioms S2–S4, the scalar profile is uniquely determined as $f(u) = \ln(1 + (N-1)u)$. Relative to S1, the scalar axioms S2, S3, and S4 are each independent.
- Monotonicity and concavity are consequences, not assumptions.
- The surplus decomposes into crossing and divergence components.
- The surplus has sharp global Lipschitz constant $N - 1$ and is extremized by parallel/zero and orthogonal/maximum pairs.

Acknowledged:

- S3 (affine effective count) is the design axiom that does the characterization work. It is justified as the minimal parameter-free effective-count interpolation compatible with the endpoint constraints, not derived from more primitive hypotheses.
- The triangle inequality fails for geometric triples; subadditivity holds only on the parameter u.
- All claims are internal to the formal system. No claims about physical reality, consciousness, or ontology are made here.
- This paper presents pure mathematics; falsifiability is not applicable as there are no empirical claims. The theorems are proved from axioms.

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This paper contains only definitions, axioms, theorems, and proofs. No empirical or interpretive claims are used anywhere in the arguments.

Future work. The direct-sum structure used here also appears in block-structured signal, representation, and computational settings. Whether surplus functionals of the present type are useful there is an open question; no such application is proved in this paper.

Cross-Domain Interaction as a Formal Modeling Framework: Interpreting the Surplus Functional

Paper B: Modeling Interpretation

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Abstract. Paper A proved that the surplus functional $f(u) = \ln(1 + (N-1)u)$ is uniquely characterized by four axioms (S1–S4) on a block-decomposed vector space. This paper asks what kinds of target domains can be represented by that formal structure without extending its mathematics. We propose a constrained modeling interpretation in which blocks represent idealized categories, unit vectors represent normalized states, and surplus measures modeled accessible-structure gain generated by difference. The paper makes the representation map explicit, states validity conditions and failure cases, treats Variable X as an optional extension rather than a foundation, and locates the proposal relative to structural realism, formal ontology, philosophy of information, and model-based philosophy of science. It is not an empirical test and does not claim to reveal the literal structure of reality.

Keywords. formal ontology, structural realism, philosophy of information, model-based philosophy of science, interaction surplus

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0. Axiom Restatement

For reference, Paper A's four axioms (S1–S4) that this paper interprets:

S1 *(Geometric dependence).* $F(\vec{x}, \vec{y}) = f(u(\vec{x}, \vec{y}))$ depends only on bivector norm u.

S2 *(Zero at zero).* $f(0) = 0$.

S3 *(Affine effective count).* $g(u) = e^{f(u)}$ is affine: $g(u) = 1 + (N-1)u$.

S4 *(Normalization).* $f(1) = \ln N$.

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1. The Modeling Framework

1.1 From Algebra to Representation

Paper A defines: - $V = \mathbb{R}^{NM}$ with N orthogonal blocks $\{H_i\}$ of dimension M. - Surplus functional $f(u) = \ln(1 + (N-1)u)$ on unit vector pairs.

We assign interpretive content:

Formal object	Interpretation
Block H_i	An *idealized category* in a chosen representation
Unit vector $\vec{x} \in H_i$	A *normalized state-description* within category H_i
Bivector norm $u = \|\vec{x} \wedge \vec{y}\|^2$	*Represented difference*: 0 = parallel/equivalent in the model, 1 = orthogonal/maximally distinct in the model
Surplus $f(u)$	*Modeled accessible-structure gain* produced by coupling two represented states
$u_{\text{cross}} = \beta^2$	Between-category contribution to represented difference
$u_{\text{div}} = \alpha^2 \sin^2 \gamma$	Within-category divergence contribution to represented difference

1.2 Validity Conditions

The modeling map applies only after the target domain has been given an explicit encoding into the formal structure. Concretely, one must specify:

1. **State encoding:** A map from target-domain items to normalized vectors, so that each represented state has a determinate location in the model.
2. **Category assignment:** A finite partition into idealized categories, represented by blocks $H_1, \ldots, H_N$.
3. **Geometric dependence** (S1 analogue): Interaction must be modeled through a scalar similarity parameter such as an angle, cosine, or inner-product-derived quantity.
4. **Zero baseline** (S2 analogue): Parallel or representationally equivalent states should correspond to zero modeled surplus.
5. **Affine effective-count law** (S3 analogue): The effective-count interpretation is adopted as a modeling assumption, not read directly off the world.
6. **Normalization** (S4 analogue): The maximum modeled surplus is finite and depends on the chosen category count N.

Note: Monotonicity and concavity are *derived* in Paper A (Theorems 3.1, 3.3), not axiomatic.

A target domain that lacks a stable encoding, a defensible category scheme, or an interpretable angle-like geometry is outside the scope of this model.

1.3 Representational Limits

The present formalism does not, by itself, represent:

- overlapping or mixed-membership categories without a non-orthogonal extension of Paper A,
- asymmetric interactions where x acting on y differs essentially from y acting on x,

- temporal or causal structure beyond static pairwise interaction,
- domains for which no reasonable inner-product-like representation is available.

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2. Illustrative Corpus Encoding

2.1 The Specific Model

The *Vovina Ontological OmniTautology* corpus (500 volumes, 72M words; Curzi, 2026) can be encoded as one illustrative instance of the formal system with $N = 12$, $M = 12$:

i	House	Domain
1	Mathematics & Pattern	Formal structure, number, proof
2	Physics & Energy	Matter, force, measurement
3	Biology & Body	Life, organism, genetics
4	Psychology & Mind	Cognition, emotion, development
5	Consciousness & Field	Awareness, observation, qualia
6	Communication & Code	Value, exchange, accounting
7	Technology & Craft	Tools, systems, engineering
8	Spirituality & Practice	Contemplation, tradition, experience
9	Art & Expression	Aesthetics, creation, performance
10	Ecology & Earth	Ecosystem, soil, climate
11	Language & Story	Grammar, narrative, translation
12	Ethics & Governance	Law, sovereignty, distribution

- $M = 12$ sub-modes ("rooms") per category: Origin, Structure, Flow, Perception, Reciprocity, Transformation, Harmony, Growth, Memory, Repair, Connection, Completion.
- **Optional Variable X:** if one wants a distinguished probe that touches all chosen categories at once, it can be modeled as the diagonal state $\hat{e}_X = \frac{1}{\sqrt{12}} \sum_i e_{i1}$ spanning the room-1 directions across all categories.

2.2 Interpretive Propositions (Derived from Paper A)

The following interpretive propositions are logically derived from Paper A's theorems once the representation has been fixed. Propositions B.1–B.3 do not depend on Variable X. Proposition B.4 is optional and concerns only the special diagonal construction.

Proposition B.1 *(Non-parallel generates surplus).* For any two non-parallel states, the interaction surplus is strictly positive.

Proof. By Paper A, Theorem 2.1, $f(u) = \ln(1 + (N-1)u)$. By Theorem 3.2, $f(u) > 0$ for all $u > 0$. Therefore non-parallel states (having $u > 0$) generate positive surplus. □

Proposition B.2 *(Decomposition into crossing and divergence).* Information gain decomposes into contributions from category crossing and within-category divergence.

Proof. Direct application of Paper A, Theorem 4.1. Therefore $F = f(u_{\text{cross}} + u_{\text{div}})$ with bounded contributions from each source. □

Proposition B.3 *(Orthogonality maximizes surplus).* Total information gain is maximized by orthogonal pairs; in particular, pure states drawn from distinct orthogonal blocks achieve this maximum.

Proof. By Paper A, Theorem 4.3, $F = \ln N$ iff $\vec{x} \perp \vec{y}$. Orthogonal pairs therefore maximize surplus, and distinct orthogonal blocks instantiate this maximum. □

Proposition B.4 *(Optional diagonal probe).* If one defines the diagonal state $\hat{e}_X$, then it generates positive surplus with every pure block state.

Proof. By Paper A, Theorem 4.4, $F(\vec{x}, \hat{e}_X) > 0$ for all $\vec{x} \in H_i$. Thus any distinguished diagonal probe of this form interacts positively with every pure block state. This construction is available, but not required for the rest of the model. □

2.3 What the Model Does Not Claim

- **Number of categories:** 12 is not asserted as "correct." Other decompositions ($N = 7$, $N = 20$, etc.) produce different maxima but same structural theorems.
- **Exhaustive taxonomy:** The *Vovina Ontological OmniTautology* partition is one idealized coding, not a uniquely privileged carving of the world.
- **Physical orthogonality:** Block orthogonality in Paper A is a simplifying assumption. Real domains overlap (physics/mathematics share methods; biology/psychology share the nervous system study).
- **Physical realization:** Any measurable interpretation of surplus is domain-specific and empirical, not established by this paper.
- **Metaphysical truth:** The formal system is a *useful model* for cross-domain reasoning, not claimed as the literal structure of reality.
- **Triangle inequality:** Paper A §5 explicitly shows the triangle inequality *fails* for geometric triples. The model makes no metric space claims.

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3. Central Modeling Thesis

3.1 Core Claim

The illustrative corpus encoding above instantiates one recurring structural pattern:

> **When two represented states are sufficiently different, the model assigns positive surplus. Cross-category difference and within-category divergence are the two structural modes by which that difference can occur.**

Paper A formalizes this as: $f(u) > 0$ for $u > 0$ (non-parallel states yield surplus).

The restrained philosophical claim is therefore not that interaction is metaphysically fundamental, but that the formalism is well-suited to representing situations in which difference is modeled as productive rather than merely classificatory. This is a claim about the use of the representation, not a proof about what reality is made of.

3.2 Interpretive Alignment

A target domain is representable by this model only insofar as: 1. States admit an explicit encoding into a vector-like representation. 2. Interaction depends on a scalar similarity parameter in that

representation. 3. Zero surplus corresponds to parallel or equivalent states in that representation. 4. Effective accessible-state count is modeled as affine in that parameter (design axiom corresponding to S3). 5. Maximal surplus is finite and fixed by the chosen category count (normalization).

3.3 The Two-Source Thesis

The decomposition theorem (Paper A, Theorem 4.1) yields two interpretive components:

1. **Widening** (crossing surplus): engaging a *different kind* of thing opens new categories of possibility.
2. **Deepening** (divergence surplus): engaging a *different angle* on the same kind opens new directions within existing possibility.

Both are real. Both generate surplus. Therefore a complete account of knowledge generation requires both widening and deepening. The difference is structural: - Widening adds *new dimensions* to accessible space. - Deepening adds *new directions* within existing dimensions.

Thus the modeling thesis is: **difference generates modeled surplus, with crossing as widening and within-domain divergence as deepening.**

3.4 Comparison with Frameworks

This proposal is closest to four existing literatures. From **structural realism** (Worrall, 1989; Ladyman & Ross, 2007), it borrows the idea that relational structure can carry explanatory weight. From **formal ontology** (Guarino, 1998), it borrows the demand that categories and admissible assignments be explicitly typed rather than informally named. From the **philosophy of information** (Floridi, 2011), it borrows the idea that informational difference can be a useful explanatory variable. From **model-based philosophy of science** (Giere, 1988), it borrows the thought that an idealized formal structure can be illuminating without being literally true.

The present paper therefore should be read as a partial representational scheme. It is more committal than a loose metaphor, but less committal than a full ontology.

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4. Optional Role of Variable X

Variable X is not needed for the core modeling framework. If one wants a distinguished probe that touches all chosen categories at once, it can be modeled as the diagonal state $\hat{e}_X$.

Formally, $\hat{e}_X$ generates positive surplus with every non-diagonal pure state (Paper A, Theorem 4.4). Interpretively:

> A probe distributed across all chosen categories cannot be parallel to a pure single-category state.

This is the formal content of the optional diagonal construction. The framework and its main interpretive propositions stand without it.

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5. Limitations and Open Problems

5.1 Orthogonality Assumption

Block orthogonality is the strongest simplifying assumption. Real domains overlap. An affinity matrix $G \neq I$ could capture this, but requires empirical/structural input to specify G.

5.2 Choice of N and M

The formal system works for any $N, M \geq 2$. The *Vovina Ontological OmniTautology* corpus uses $N = M = 12$. Whether 12 is the "right" number of ontological categories is philosophical, not mathematical. The formal theorems hold for all N.

5.3 Meaning of "Surplus"

In the formal system, surplus is a real number measuring state-space expansion. In the interpretation, it is glossed as "information gain." Whether this maps to anything measurable is empirical (Paper C).

5.4 Structural Resonances with Computational Frameworks

This subsection identifies three structural analogies between Paper A's formal system and frameworks in parallel/distributed computation and optimization. These are *modeling analogies*—they clarify the model's structure and locate it within a broader mathematical landscape. They are not proof ingredients: none of the results below are used to establish any theorem in Paper A, and they carry no empirical weight toward Paper C's predictions.

Decomposition and consistency. Paper A's ambient space decomposes as $V = H_1 \oplus H_2 \oplus \cdots \oplus H_N$, with surplus measured across the decomposition. This is formally identical in structure to the direct-sum decompositions used in parallel simulation architectures, where a composite state space is partitioned into sub-simulators that operate concurrently and must remain mutually consistent. In those settings, a key correctness condition is *interaction preservation*: the behavior of the composed system must be recoverable from the behaviors of its parts together with their interfaces. Paper A's Decomposition Theorem (Theorem 4.1) is the formal analogue: surplus $f(u)$ is recoverable from the crossing and divergence components $(u_{\text{cross}}, u_{\text{div}})$, with precise bounds. The analogy is structural, not causal; it suggests that the interaction-surplus model may be naturally expressible in a compositional, modular idiom.

Convergence and iterative interaction. The uniqueness result of Paper A (Theorem 2.1) is reminiscent of fixed-point uniqueness results in iterative convergence theory. In distributed consistency protocols, the Banach contraction mapping theorem guarantees that a unique fixed point is reached by repeated application of a contracting update rule. The analogy here is: the axioms S1–S4 function as a "contraction" on the space of candidate surplus functionals, eliminating all alternatives and converging to the unique $f(u) = \ln(1 + (N-1)u)$. The difference is that Paper A's uniqueness is algebraic (five lines of direct derivation), not iterative; but the structural role—that a small set of constraints uniquely determines a global object—is shared. This analogy motivates thinking of S3 in particular as the contraction condition: it is the design axiom that eliminates exponential, polynomial, and power-law alternatives, leaving exactly one function.

Optimization and estimation. Fitting a surplus model to empirical interaction data (Paper C's task) is an optimization problem: find parameters (N, M) and candidate block assignments that

minimize a discrepancy between predicted and observed surplus values, subject to the constraint that the resulting functional satisfies S1–S4. This is structurally analogous to multi-objective optimization with constraints: a combination of model fit (minimize residuals) and model compliance (satisfy axioms). Gradient-based methods apply when surplus predictions are differentiable in model parameters; Pareto-optimal trade-off analysis applies when multiple goodness-of-fit criteria compete. The formal surplus functional $f(u) = \ln(1 + (N-1)u)$ provides the theoretical curve against which empirical surplus estimates are benchmarked. These are implementation-level modeling choices developed in Paper C; they are noted here only to illustrate that the formal system admits natural estimation counterparts.

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6. Summary

Question	Answer
What does Paper B do?	Proposes a constrained interpretation of Paper A's formal system as a model of structured interaction (cross-domain and within-domain divergence)
Is the interpretation provable?	No. Interpretations are proposed, tested, and evaluated for usefulness—not proved
What does the model predict?	Non-trivial interactions generate surplus; surplus is maximal for maximally distinct domains; surplus decomposes into crossing and divergence
What does the model not predict?	Specific number of categories; physical reality of surplus as energy; literal metaphysical truth
What is the central thesis?	In any valid representation, difference generates modeled surplus; crossing and divergence are the two main structural modes.

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This paper offers a modeling interpretation of Paper A's formal system, stated precisely enough to be evaluated, criticized, and refined.

Empirical Predictions of the Interaction Surplus Framework

Paper C: Empirical Bridge

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Abstract. Paper A proved that the surplus functional $f(u) = \ln(1 + (N-1)u)$ is uniquely characterized by four axioms (S1–S4) on a block-decomposed vector space. This paper turns the empirical bridge into one preregistration-ready flagship test in transformer neural networks. Conditional on the Bridge Principle (BP), we test whether edge-level predictive gain in GPT-2 small is better fit by $c \ln(1 + \lambda \sin^2 \theta)$ than by linear, quadratic, power-law, exponential, and standard attention-based baselines. We specify the exact dataset split, representation, observable, ablation estimator, model-comparison rule, falsification criterion, and code-release requirements. Additional domains are deferred to later work. This is an empirical paper: the flagship test may succeed or fail.

Keywords. transformer attention, preregistration, model comparison, predictive gain, ablation, empirical test

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0. Axiom Restatement

For reference, the four axioms from Paper A that ground these empirical predictions:

S1 *(Geometric dependence).* $F(\vec{x}, \vec{y}) = f(u(\vec{x}, \vec{y}))$ for $u = \|\vec{x} \wedge \vec{y}\|^2 = \sin^2 \theta$.

S2 *(Zero at zero).* $f(0) = 0$.

S3 *(Affine effective count).* $g(u) = e^{f(u)} = 1 + (N-1)u$.

S4 *(Normalization).* $f(1) = \ln N$.

Derived Form: By Paper A, Theorem 2.1, the unique solution is $f(u) = \ln(1 + (N-1)u)$.

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1. The Bridge Principle

1.1 From Information to Measurement

Paper A's surplus functional $f(u) = \ln(1+(N-1)u)$ measures logarithmic gain in effective accessible states. To connect to empirical measurements, we need a *bridge principle*—a rule mapping formal

quantities to measurable ones.

Bridge Principle (BP). If a physical system's state space decomposes into N distinguishable categories, and two states have measurable "angle" θ (arccosine of normalized inner product), then information-theoretic surplus follows:

$$\Delta I = c \cdot \ln(1 + (N-1)\sin^2\theta)$$

for domain-specific constant $c > 0$ converting abstract information to native units. In the flagship transformer test below, those units are token-level negative-log-likelihood differences.

Why edge-ablation predictive gain is the operational proxy. Paper A defines surplus as the contribution of geometric difference to effective accessible structure: when two states are non-parallel, the interaction generates positive surplus proportional to the log-effective-count increase. In the transformer setting, a single attention edge connects a query position to a key position via a geometric inner-product-derived weight. Single-edge ablation measures the contribution of that specific interaction to the model's predictive performance: removing the edge and observing the change in negative log-likelihood isolates the edge's marginal contribution. Therefore, edge-level predictive gain is used as an operational proxy for modeled surplus. This is a *measurement choice*, not an identity theorem: predictive gain is not mathematically identical to Paper A's $f(u)$, but it is the closest domain-available observable that tracks the same structural quantity—contribution of pairwise geometric difference to system performance.

What BP claims: the *functional form* $\ln(1 + \lambda \sin^2\theta)$ governs interaction surplus, with $\lambda = N - 1$ determined by number of categories (per Paper A, Theorem 2.1).

What BP does not claim: the value of c, which must be fit empirically per domain. BP predicts the *shape*, not absolute scale.

1.2 Falsification of BP

BP is falsified if, in a domain satisfying categorization conditions, measured surplus follows a different functional form—e.g., linear, quadratic, or exponential in $\sin^2\theta$—with no value of λ or c fitting the data. Therefore BP provides a precise falsification criterion.

1.3 Derived Predictions

From Paper A's theorems, we derive the following empirical consequences:

- **Monotonicity** (Theorem 3.1): More different states produce more surplus.
- **Concavity** (Theorem 3.3): Diminishing returns as dissimilarity increases.
- **Maximal value** (Theorem 4.3): Maximum surplus is $\ln N$ for orthogonal states from distinct categories.

Thus the bridge principle encapsulates the empirical content of Paper A's formal system.

1.4 Statistical Evaluation Framework

- **Primary criterion:** AIC/BIC model comparison among log-affine and alternative functional forms.
- **Secondary criterion:** Out-of-sample predictive error via cross-validation.

- **Uncertainty:** Bootstrap confidence intervals for λ and c; require λ CI excluding 0 for support.
- **Robustness:** Re-fit under alternative distance definitions; require consistent log-affine advantage across resamples/specifications.
- **Failure criterion:** No stable log-affine advantage in AIC/BIC and out-of-sample error across robustness checks.

Preregistration scope. Prediction P1 is stated with sufficient specificity for preregistration. Before evaluating the held-out split, the analysis plan should lock the dataset construction rule, sequence count, layer/head selection, edge-ablation definition, clustering rule, model families, exclusions, bootstrap count, and decision thresholds. Appendix B describes a simulation environment for piloting the pipeline before preregistration lock, and Appendix D specifies the release package needed for external reproduction.

Causal scope. The flagship test includes a model-internal intervention—single-edge attention ablation—but it does not establish causal claims about cognition, language, or physical systems. The test evaluates whether the BP functional form is a better description of transformer predictive gain than its competitors. Any broader causal interpretation lies outside the scope of this paper.

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2. Flagship Empirical Test: Transformer Neural Networks

2.1 Primary Domain and Data

This paper evaluates BP in one domain only: autoregressive transformer language models. The primary model is GPT-2 small (124M parameters, 12 layers, 12 heads). The primary dataset is the public OpenWebText corpus, tokenized with GPT-2 byte-pair encoding, deterministically shuffled with seed 20260308, concatenated, and segmented into 50,000 non-overlapping sequences of length 128 tokens. The split is fixed in advance:

- **Pilot split:** first 10,000 sequences (used only for clustering and comparator lock-in)
- **Fit split:** next 20,000 sequences (used for parameter estimation)
- **Held-out test split:** final 20,000 sequences (used once for model comparison)

The primary analysis uses layer 6 and all 12 heads. No layer selection is performed after inspecting test outcomes.

2.2 Representation, Categorization, and Geometry

For each target position i and source position $j < i$ in layer 6, head h, extract the query vector $Q_i^{(h)}$ and key vector $K_j^{(h)}$. Define

$$s_{ijh} = \frac{Q_i^{(h)} \cdot K_j^{(h)}}{\|Q_i^{(h)}\| \|K_j^{(h)}\|}, \qquad u_{ijh} = 1 - s_{ijh}^2.$$

The quantity u_{ijh} is the empirical analogue of Paper A's interaction parameter.

Categories are defined on the pilot split only. L2-normalize the key vectors and fit k-means++ for $k \in \{8, 10, 12, 14, 16\}$ with 20 random restarts per k. Select N_{clusters} by the largest mean silhouette score. Lock the resulting centroids and assign every source token in the fit and test splits to its nearest centroid. This prevents post hoc category tuning.

2.3 Primary Prediction and Observable

For each edge $j \to i$ with attention weight $\alpha_{ijh} \geq 0.01$, define the edge-level predictive gain

$$G_{ijh} = \ell_i^{\text{abl}(j,h)} - \ell_i^{\text{full}},$$

where $\ell_i = -\ln p_\theta(t_i \mid t_{<i})$ is the negative log-likelihood of the target token and $\text{abl}(j,h)$ means: zero only the contribution of source position j inside head h at target position i, leaving all other activations unchanged. Positive G_{ijh} means that the edge was beneficial in the full model, since removing it worsens prediction.

P1. Conditional on BP, the mean predictive gain is better described by

$$\mathbb{E}[G_{ijh} \mid u_{ijh}] = c \ln(1 + \lambda u_{ijh}), \qquad c > 0,\ \lambda > 0,$$

than by the comparator families listed below.

Secondary theory-consistency check. Support for BP is strengthened if the bootstrap confidence interval for λ overlaps $N_{\text{clusters}} - 1$, but exact equality is not required because the empirical clustering is only an approximation to the idealized category structure.

Sub-predictions (conditional on BP):

- After matching edges by attention-weight decile, between-cluster edges have larger mean predictive gain than within-cluster edges.
- The fitted relationship in u is strictly increasing and concave on the observed range.

2.4 Comparator Models

All models are fit on the fit split and evaluated once on the held-out test split.

Geometry-only competitors (same predictor family in u):

- **Log-affine:** $c \ln(1 + \lambda u)$
- **Linear:** $a + bu$
- **Quadratic:** $a + bu + du^2$
- **Power-law:** $a + bu^k$
- **Exponential saturation:** $a + b(1 - e^{-cu})$

Transformer-specific baselines:

- **Attention-only:** $a + b\alpha_{ijh}$
- **Score-only:** $a + b\, Q_i^{(h)} K_j^{(h)} / \sqrt{d_k}$
- **Spline baseline:** cubic spline in u with 5 degrees of freedom
- **Feature baseline:** ridge regression on $[\alpha_{ijh},\ Q_i^{(h)} K_j^{(h)} / \sqrt{d_k},\ |i-j|,\ \log(\text{token frequency rank}),\ \text{head indicator}]$

Hyperparameters for the spline and ridge baselines are fixed on the pilot split and not re-tuned on the fit or test splits.

2.5 Decision Rule and Falsification Criterion

Primary support criterion. P1 is supported only if all of the following hold:

1. Among the geometry-only families, the log-affine model has the lowest BIC on the fit split with $\Delta\text{BIC} \geq 10$ against each of the linear, quadratic, power-law, and exponential competitors.
2. On the held-out test split, the log-affine model has lower RMSE than the attention-only and score-only baselines.
3. A 95% bootstrap confidence interval for λ excludes 0.

Robustness criterion. The geometry-only ranking is repeated headwise, and the log-affine model should rank first by BIC in at least 8 of the 12 heads.

Falsification. P1 is falsified if any of the three primary support criteria fail. Failure of the headwise robustness criterion weakens the claim even if the aggregate fit remains favorable.

2.6 Preregistration and Release Package

Before the held-out test split is evaluated, the following items should be frozen and released in machine-readable form: dataset construction script, shuffle seed, sequence manifest, layer/head specification, attention-edge inclusion threshold, clustering rule, comparator model list, bootstrap count, exclusion rules, and decision criteria. A publishable version of this empirical paper should also be accompanied by a public repository containing the edge-ablation code, fitting scripts, environment file, fixed random seeds, and a one-command reproduction script. Appendix D specifies the required release package and execution guarantees.

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3. Deferred Domains

Neuroscience, economics, and atomic physics are not part of the main empirical claim of this paper. They remain candidate applications of the interaction-surplus framework, but each raises harder measurement or identification problems than the transformer case. To keep the paper falsifiable and replicable, those domains are deferred. Short sketches are collected in Appendix A as future-work notes rather than as active claims.

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4. Summary of the Flagship Test

ID	Domain	Observable	Main model	Comparator standard	Falsifiable?
P1	GPT-2 small on OpenWebText	Edge-level predictive gain under single-edge ablation	$c\ln(1+\lambda u)$ with $u = 1 - \cos^2\theta$	Functional competitors + transformer baselines	Yes

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5. What This Paper Does and Does Not Do

It does:

- State one preregistration-ready flagship empirical test conditional on BP.
- Specify an exact dataset split, representation, observable, comparator family, and decision rule.
- Define the code-release and reproducibility package required for outside verification.
- Separate the main empirical claim from deferred domains.

It does not:

- Claim empirical support in neuroscience, economics, or atomic physics.
- Treat flexible or field-standard baselines as irrelevant.
- Claim that one successful transformer result would establish BP universally.
- Claim real-world causal identification beyond the model-internal edge-ablation intervention.

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6. Relationship Between Papers

Paper	Contains	Status if predictions fail
A	Axioms, definitions, theorems, proofs	Unaffected—formal validity independent of empirical truth
B	Modeling interpretation	Weakened—if no domain fits the model, usefulness is reduced
C	One flagship empirical test	Falsified—this specific test fails, requiring revision

This separation is the point. Mathematics (A) is internal to axioms. Modeling interpretation (B) is evaluated for representational usefulness. Empirical testing (C) is answerable to data.

Appendix A: Deferred Domain Sketches

A.1 Neuroscience

A future neuroscience test would require a defensible analogue of the geometric parameter u, a stable definition of category structure across brain regions, and a low-bias information estimator at the neuron-pair level. The present paper does not claim that these ingredients are already in place.

A.2 Economics

A future economics test would need a causal design capable of addressing endogeneity in partnership formation. Event-study or panel-regression evidence may still be useful, but the measurement and identification burden is much heavier than in the transformer case.

A.3 Atomic Physics

The atomic-physics discussion is best understood as a structural analogy. It may motivate future formal comparisons, but it is not part of the present empirical claim and is not treated here as a flagship test.

Appendix B: Simulation Environment for Transformer Protocol Piloting

This appendix describes a synthetic simulation environment for piloting the transformer protocol before preregistration lock or access to the held-out split. The purpose is to verify that the decision rule in §2.5 can recover the log-affine law when it is the ground truth and reject it when it is not.

Synthetic data generation. Draw $u_k \sim \mathrm{Beta}(2,2)$ on $[0,1]$ and generate edge-level gains by

$$G_k = c_0 \ln(1 + \lambda_0 u_k) + \varepsilon_k, \qquad \varepsilon_k \sim \mathcal{N}(0, \sigma^2).$$

To mimic transformer covariates, also generate synthetic attention weights α_k and positional distances d_k with moderate correlation to u_k. This allows the feature baselines in §2.4 to be stress-tested against realistic confounding structure.

Deterministic execution. Every simulation run is seeded before sampling. The seed, parameter tuple $(c_0, \lambda_0, \sigma^2)$, and sample size are written to disk with the result file. Re-running with the same metadata reproduces the same synthetic dataset exactly.

Calibration target. Across 200 replications, the support criteria in §2.5 should identify the log-affine model in at least 90% of runs when it is the ground truth and reject it in at least 90% of runs when the ground truth is linear or quadratic. If not, the protocol should be revised before preregistration.

Appendix C: Optimization and Comparator Fitting

C.1 Log-Affine Parameter Fitting

Given observed pairs (u_k, G_k) from the fit split, estimate (c, λ) by nonlinear least squares:

$$\mathcal{L}(c, \lambda) = \sum_{k=1}^{n} \left(G_k - c \ln(1 + \lambda u_k)\right)^2,$$

subject to $c > 0$, $\lambda > 0$. Use projected gradient descent with Armijo backtracking and initialize from the grid

$$c_0 \in \{0.1, 0.5, 1.0, 2.0\}, \qquad \lambda_0 \in \{0.5, 1.0, 2.0, 4.0, 8.0\}.$$

The bootstrap interval for λ is computed from 1,000 resamples of the fit split.

C.2 Cluster Selection

The cluster count is selected on the pilot split only. For each $k \in \{8, 10, 12, 14, 16\}$, run k-means++ with 20 random restarts and record the mean silhouette score. Lock the k with the highest score before any model fitting on the fit or test splits.

C.3 Comparator Families

Linear, quadratic, power-law, and exponential-saturation models are fit by least squares or nonlinear least squares on the fit split. The spline model uses 5 degrees of freedom fixed in advance. Ridge regression uses regularization strength chosen on the pilot split only. BIC is computed on fit-split residual variance, and RMSE is reported on the held-out test split.

Appendix D: Code Release, Distributed Execution, and Reproducibility

D.1 Required Public Release Package

A publishable release of the flagship test should include:

- dataset construction script and sequence manifest
- exact shuffle seed and split definitions
- edge-ablation implementation
- clustering and fitting scripts
- environment lockfile and package versions
- bootstrap and evaluation scripts
- one-command reproduction entry point

D.2 Parallel Execution

The P1 pipeline is naturally parallel over heads, bootstrap draws, and fit initializations. These tasks are independent until the final aggregation step, so running them on multiple cores or machines does not change the result provided the seeds are fixed.

D.3 Reproducibility Protocol

All stochastic steps use a two-level seed scheme: a global seed s_0 and task-specific derived seeds $s_t = H(s_0, t)$. Every result file should log s_0, the task identifier, software versions, hardware metadata, and checksums for the sequence manifest.

D.4 Compute Requirements

For the primary configuration (50,000 sequences, layer 6, 12 heads, 1,000 bootstrap resamples), the full pipeline is within standard research-computing range: parallel execution on 8 cores is sufficient, and no specialized hardware beyond the base GPT-2 inference device is required.

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These predictions may be wrong. If they are, Paper A remains valid; Paper B would need revision; Paper C would be falsified. That is how science works. Mathematics does not depend on physics. Physics does not depend on metaphysics. Each stands on its own terms.

Interaction Surplus, Effective Counts, and Information-Theoretic Bridge Functionals

Paper D: Information-Theoretic Bridge

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Abstract. Paper A characterized the surplus functional

$$F(\vec{x},\vec{y}) = \ln\big(1 + (N-1)u(\vec{x},\vec{y})\big), \qquad u(\vec{x},\vec{y}) = 1 - (\vec{x}\cdot\vec{y})^2,$$

on the unit sphere of a block-decomposed Euclidean space. The present paper studies what can and cannot be established when that surplus law is embedded into information-theoretic constructions. We prove four precise bridge results. First, surplus is exactly the logarithm of the affine effective-count law $g(u) = 1 + (N-1)u$. Second, it admits a Bernoulli moment representation:

$$F(u) = \ln \mathbb{E}[N^{B_u}], \qquad B_u \sim \text{Bernoulli}(u).$$

Third, for the canonical N-state distribution

$$P_u = \left(\frac{1}{g(u)}, \frac{u}{g(u)}, \ldots, \frac{u}{g(u)}\right),$$

its Shannon entropy satisfies an exact identity

$$H(P_u) = F(u) - \frac{(N-1)u}{g(u)} \ln u,$$

hence $H(P_u) \geq F(u)$ on $[0,1]$. Fourth, for random interaction parameters $U \in [0,1]$, Jensen bounds imply

$$\mathbb{E}[F(U)] \leq F(\mathbb{E}[U]) = \ln\bigl(1 + (N-1)\mathbb{E}[U]\bigr).$$

These theorems place the surplus functional in exact relation to effective-count and canonical entropy constructions. They do not prove thermodynamic negentropy or any physical entropy law.

Keywords. effective count, Shannon entropy, KL divergence, Rényi scale, interaction surplus

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1. Scope and Standing Assumptions

Throughout, $N \geq 2$ is fixed and all logarithms are natural. Paper A proved that under axioms S1–S4 the unique scalar surplus profile is

$$F(u) := \ln\bigl(1 + (N-1)u\bigr), \qquad u \in [0,1].$$

This paper does not alter that characterization. Its purpose is narrower: to construct exact information-theoretic objects canonically associated with F and to determine what relations are actually provable.

Two restrictions are important.

1. This paper studies information-theoretic bridges only. It makes no thermodynamic claim.
2. Any use of the word "entropy" below refers to explicitly defined information-theoretic quantities such as Shannon entropy or KL divergence.

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2. The Effective-Count Bridge

Define the effective-count map

$$g(u) := e^{F(u)} = 1 + (N-1)u.$$

Thus $g : [0,1] \to [1,N]$ is affine, with $g(0) = 1$ and $g(1) = N$.

Theorem 2.1 *(Exact effective-count representation).* The surplus law satisfies

$$F(u) = \ln g(u), \qquad g(u) = 1 + (N-1)u,$$

and g is the unique affine map from $[0,1]$ to $[1,N]$ compatible with the endpoint conditions.

Proof. The identity $F(u) = \ln g(u)$ is immediate from the definition of g. If $h(u) = au + b$ is affine with $h(0) = 1$ and $h(1) = N$, then $b = 1$ and $a = N - 1$, hence $h = g$. □

Corollary 2.2 *(Range and endpoint structure).* For all $u \in [0,1]$,

$$0 \le F(u) \le \ln N,$$

with equality on the left iff $u = 0$ and equality on the right iff $u = 1$.

Proof. Since $g([0,1]) = [1, N]$, applying the increasing map ln gives $F([0,1]) = [0, \ln N]$. The endpoint equalities follow from strict monotonicity of F from Paper A. □

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3. Distributional Bridges

3.1 A Bernoulli moment representation

Theorem 3.1 *(Bernoulli moment representation).* For each $u \in [0,1]$, let $B_u \sim \text{Bernoulli}(u)$. Then

$$F(u) = \ln \mathbb{E}[N^{B_u}].$$

Proof. Since B_u takes the value 1 with probability u and 0 with probability $1-u$,

$$\mathbb{E}[N^{B_u}] = (1-u)N^0 + uN^1 = 1 - u + uN = 1 + (N-1)u = g(u).$$

Taking logarithms yields

$$F(u) = \ln g(u) = \ln \mathbb{E}[N^{B_u}].$$

□

Remark 3.2. Theorem 3.1 gives a precise probabilistic representation, but it is not yet a mutual-information theorem. It identifies surplus with a log-moment quantity, not with Shannon entropy or KL divergence by itself.

3.2 A canonical N-state distribution

For $u \in (0,1]$, define

$$P_u := \left(\frac{1}{g(u)}, \frac{u}{g(u)}, \ldots, \frac{u}{g(u)} \right) \in \mathbb{R}^N,$$

with one distinguished coordinate equal to $1/g(u)$ and the remaining $N-1$ coordinates equal to $u/g(u)$. Since

$$\frac{1}{g(u)} + (N-1)\frac{u}{g(u)} = 1,$$

this is a probability vector. At $u = 0$, define $P_0 := (1, 0, \ldots, 0)$.

Theorem 3.3 *(Exact Shannon entropy identity).* Let $H(P) := -\sum_i p_i \ln p_i$. Then for $u \in (0,1]$,

$$H(P_u) = F(u) - \frac{(N-1)u}{g(u)} \ln u.$$

By continuity, $H(P_0) = 0$.

Proof. Write $g = g(u)$. Then

$$H(P_u) = -\frac{1}{g} \ln \frac{1}{g} - (N-1)\frac{u}{g} \ln \frac{u}{g}.$$

Expanding the logarithms gives

$$H(P_u) = \frac{1}{g} \ln g + (N-1)\frac{u}{g}(\ln g - \ln u).$$

Because $\frac{1}{g} + (N-1)\frac{u}{g} = 1$, this simplifies to

$$H(P_u) = \ln g - \frac{(N-1)u}{g} \ln u.$$

Since $F(u) = \ln g(u)$, the claimed identity follows. For $u \downarrow 0$, the factor $u \ln u \to 0$, so $H(P_u) \to 0$, which agrees with $H(P_0) = 0$. □

Corollary 3.4 *(Entropy dominates surplus on the canonical family).* For all $u \in [0, 1]$,

$$H(P_u) \geq F(u),$$

with equality iff $u \in \{0, 1\}$.

Proof. By Theorem 3.3,

$$H(P_u) - F(u) = -\frac{(N-1)u}{g(u)} \ln u.$$

For $u \in (0, 1)$, we have $\ln u < 0$, so the right-hand side is strictly positive. At $u = 0$ and $u = 1$, it vanishes by continuity and direct substitution. □

Proposition 3.5 *(Monotonicity of canonical Shannon entropy).* The function $u \mapsto H(P_u)$ is strictly increasing on $[0, 1]$.

Proof. For $u \in (0, 1)$, let $\lambda = N - 1$ and $g(u) = 1 + \lambda u$. From Theorem 3.3,

$$H(P_u) = \ln g(u) - \frac{\lambda u}{g(u)} \ln u.$$

Since

$$\left(\frac{\lambda u}{g(u)}\right)' = \frac{\lambda}{g(u)^2},$$

we obtain

$$H'(u) = \frac{\lambda}{g(u)} - \frac{\lambda}{g(u)^2}\ln u - \frac{\lambda}{g(u)} = -\frac{\lambda \ln u}{g(u)^2} > 0,$$

because $\ln u < 0$. Thus $H(P_u)$ is strictly increasing on $(0,1)$, and the endpoint values follow by continuity. □

3.3 Relative entropy to the uniform law

Let $U_N = (1/N, \dots, 1/N)$ denote the uniform distribution on N states.

Theorem 3.6 *(KL divergence to uniform).* For the canonical family P_u,

$$D(P_u \| U_N) = \ln N - H(P_u).$$

Consequently,

$$D(P_u \| U_N) \le \ln N - F(u).$$

Proof. For any distribution P on N states,

$$D(P\|U_N) = \sum_{i=1}^{N} p_i \ln \frac{p_i}{1/N} = \sum_{i=1}^{N} p_i \ln p_i + \ln N = \ln N - H(P).$$

Apply this with $P = P_u$. The inequality follows from Corollary 3.4. □

Corollary 3.7 *(Endpoint behavior).* The canonical family interpolates between maximal divergence and zero divergence:

$$D(P_0\|U_N) = \ln N, \qquad D(P_1\|U_N) = 0.$$

Proof. At $u = 0$, $P_0 = (1, 0, \dots, 0)$ has entropy 0. At $u = 1$, $P_1 = U_N$ has entropy $\ln N$. Apply Theorem 3.6. □

---

4. Random Interaction Parameters

The previous constructions are pointwise in u. We now allow U to be a random variable with values in $[0,1]$.

Theorem 4.1 *(Expected surplus upper bound).* For every random variable $U : [\Omega, \mathcal{F}, \mathbb{P}] \to [0,1]$,

$$\mathbb{E}[F(U)] \le F(\mathbb{E}[U]) = \ln\bigl(1 + (N-1)\mathbb{E}[U]\bigr).$$

Proof. Paper A proved that F is strictly concave on $[0,1]$. Jensen's inequality for concave functions therefore gives

$$\mathbb{E}[F(U)] \leq F(\mathbb{E}[U]).$$

Substituting the explicit form of F yields the stated identity. □

Corollary 4.2 *(Expected effective count identity).* For every random $U \in [0,1]$,

$$\mathbb{E}[g(U)] = 1 + (N-1)\mathbb{E}[U].$$

Hence

$$\exp(\mathbb{E}[F(U)]) \leq \mathbb{E}[g(U)].$$

Proof. The first identity holds because g is affine. The second follows by exponentiating Theorem 4.1. □

Remark 4.3. Corollary 4.2 is the cleanest distributional statement available at the level of Paper A's formalism: expected surplus is controlled by the log of the expected effective count. No stronger identification with Shannon mutual information follows without further structural assumptions.

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5. What This Paper Establishes and What It Does Not

Established exactly.

- Surplus is the logarithm of a unique affine effective-count law.
- Surplus has an exact Bernoulli moment representation.
- Surplus is a lower bound for the Shannon entropy of the canonical family P_u.
- Residual divergence to the uniform law is bounded above by $\ln N - F(u)$.
- For random interaction parameters, expected surplus is bounded by the surplus at the expected interaction level.

Not established here.

- No theorem identifies surplus with Shannon entropy in general.
- No theorem identifies surplus with mutual information in general.
- No theorem here proves thermodynamic entropy decrease, negative entropy generation, or any physical second-law statement.
- No theorem here addresses black holes, cosmology, or modified arithmetic.

The mathematically honest conclusion is therefore the following: Paper A's surplus functional admits exact effective-count and canonical information-theoretic embeddings, but these bridges do not by themselves prove a thermodynamic negentropy law.

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This paper is a bridge paper. Every theorem is internal to explicit information-theoretic constructions built from Paper A. No empirical or thermodynamic claim is proved here.

Open-System Surplus Dynamics, Candidate Negentropy Budgets, and Engineering Test Criteria

Paper E: Dynamics and Open-System Framework

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Abstract. Paper A supplied a static surplus law, and Paper D established exact bridges from that law to effective-count and canonical information-theoretic quantities. The present paper studies what can be formalized at the dynamical level without overclaiming. We first isolate a canonical concentration functional

$$\mathcal{N}_{\mathrm{can}}(u) := D(P_u \| U_N),$$

where P_u is the canonical distribution from Paper D and U_N is the uniform distribution on N states. We prove that $\mathcal{N}_{\mathrm{can}}$ is strictly decreasing in u, so the static surplus variable $F(u)$ cannot itself be identified with canonical negentropy. We then introduce a separate open-system storage variable Q_t driven by surplus input through the exact discrete-time law

$$Q_{t+1} = (1-\delta)Q_t + \eta F(U_t) - C_t,$$

with loss rate $\delta \in [0,1]$, conversion efficiency $\eta \geq 0$, interaction level $U_t \in [0,1]$, and nonnegative cost C_t. For this framework we prove a closed-form solution, a one-step growth criterion, boundedness under bounded forcing, a steady-state formula under constant input, and an exact feasibility condition for engineering control. We then state preregistration-ready testing criteria and a minimal engineering specification. No theorem here establishes a universal physical negentropy law.

Keywords. dynamical systems, open systems, KL divergence, system identification, order budget, interaction surplus

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1. Scope and Conceptual Distinction

Paper A is static: it characterizes the scalar law

$$F(u) = \ln\bigl(1 + (N-1)u\bigr), \qquad u \in [0,1].$$

Paper D showed that F has exact effective-count and canonical entropy embeddings. Those embeddings are useful, but they do not by themselves produce a dynamical law.

This paper therefore distinguishes two different objects.

1. **Canonical concentration**: an instantaneous information-theoretic quantity derived from Paper D.
2. **Stored order**: a separate state variable Q_t intended to model retained organization in an open system.

The distinction is necessary. The first quantity is defined exactly from the canonical family P_u and decreases as interaction becomes maximally mixing. The second is a dynamical variable introduced by explicit modeling assumptions. Conflating them would overstate what follows from Paper A and Paper D.

---

2. Canonical Concentration and the Limits of Static Negentropy Claims

Let $\lambda := N - 1$, let

$$g(u) := 1 + \lambda u,$$

and let the canonical distribution from Paper D be

$$P_u = \left(\frac{1}{g(u)}, \frac{u}{g(u)}, \ldots, \frac{u}{g(u)}\right), \qquad u \in [0,1].$$

Let $U_N = (1/N, \ldots, 1/N)$ denote the uniform distribution. Define the canonical concentration functional

$$\mathcal{N}_{\mathrm{can}}(u) := D(P_u \| U_N) = \ln N - H(P_u).$$

Theorem 2.1 *(Exact formula and monotonicity).* For all $u \in (0,1]$,

$$\mathcal{N}_{\mathrm{can}}(u) = \ln N - \ln g(u) + \frac{\lambda u}{g(u)} \ln u.$$

Moreover, $\mathcal{N}_{\mathrm{can}}$ extends continuously to $u = 0$ with $\mathcal{N}_{\mathrm{can}}(0) = \ln N$, satisfies $\mathcal{N}_{\mathrm{can}}(1) = 0$, and is strictly decreasing on $[0,1]$.

Proof. Paper D proved that

$$H(P_u) = \ln g(u) - \frac{\lambda u}{g(u)} \ln u.$$

Therefore

$$\mathcal{N}_{\mathrm{can}}(u) = \ln N - H(P_u) = \ln N - \ln g(u) + \frac{\lambda u}{g(u)} \ln u.$$

Continuity at $u = 0$ follows from $u \ln u \to 0$ and $g(0) = 1$. At $u = 1$, $P_1 = U_N$, so $\mathcal{N}_{\mathrm{can}}(1) = 0$. Paper D also showed that $H(P_u)$ is strictly increasing on $(0, 1)$, hence $\mathcal{N}_{\mathrm{can}}(u) = \ln N - H(P_u)$ is strictly decreasing there. □

Corollary 2.2 *(No direct surplus = canonical negentropy identification).* For all $u \in [0, 1]$,

$$F(u) + \mathcal{N}_{\mathrm{can}}(u) \leq \ln N,$$

with equality iff $u \in \{0, 1\}$.

Proof. By Paper D, $H(P_u) \geq F(u)$ for all $u \in [0, 1]$, with equality only at $u = 0$ and $u = 1$. Since $\mathcal{N}_{\mathrm{can}}(u) = \ln N - H(P_u)$,

$$F(u) + \mathcal{N}_{\mathrm{can}}(u) = \ln N - (H(P_u) - F(u)) \leq \ln N,$$

with equality exactly at the endpoints. □

Remark 2.3. Corollary 2.2 gives a rigorous warning. Higher static surplus does not imply higher canonical concentration. Thus any open-system "negentropy" interpretation requires additional dynamical structure beyond Paper A and Paper D.

---

3. A Minimal Open-System Storage Model

We now introduce a separate stored-order variable.

3.1 Definitions

Time is discrete: $t = 0, 1, 2, \ldots$. Let

- $U_t \in [0, 1]$ be the interaction parameter at time t,
- $S_t := F(U_t) = \ln(1 + (N-1)U_t)$ be the surplus input,
- $Q_t \geq 0$ be stored order,
- $C_t \geq 0$ be nonnegative cost or dissipation beyond baseline leakage,
- $\delta \in [0, 1]$ be the retention-loss parameter,
- $\eta \geq 0$ be the surplus-to-storage conversion efficiency.

We say that (Q_t, U_t, C_t) is an **admissible surplus-driven storage process** if

$$Q_{t+1} = (1 - \delta)Q_t + \eta S_t - C_t$$

for all $t \geq 0$, together with the admissibility condition

$$C_t \leq (1-\delta)Q_t + \eta S_t,$$

which guarantees $Q_{t+1} \geq 0$ whenever $Q_t \geq 0$.

3.2 Exact solution and one-step growth law

Theorem 3.1 *(Variation-of-constants formula).* For every surplus-driven storage process,

$$Q_t = (1-\delta)^t Q_0 + \sum_{k=0}^{t-1} (1-\delta)^{t-1-k}(\eta S_k - C_k), \qquad t \geq 1.$$

Proof. The statement is immediate for $t = 1$. Assume it holds at time t. Then

$$\begin{aligned} Q_{t+1} &= (1-\delta)Q_t + \eta S_t - C_t \\ &= (1-\delta)\left[(1-\delta)^t Q_0 + \sum_{k=0}^{t-1}(1-\delta)^{t-1-k}(\eta S_k - C_k)\right] + \eta S_t - C_t \\ &= (1-\delta)^{t+1} Q_0 + \sum_{k=0}^{t}(1-\delta)^{t-k}(\eta S_k - C_k), \end{aligned}$$

which is the desired formula with t replaced by $t+1$. □

Corollary 3.2 *(One-step growth criterion).* For every t,

$$Q_{t+1} - Q_t = -\delta Q_t + \eta S_t - C_t.$$

Hence

$$Q_{t+1} > Q_t \iff \eta S_t > \delta Q_t + C_t,$$

and

$$Q_{t+1} \geq Q_t \iff \eta S_t \geq \delta Q_t + C_t.$$

Proof. Subtract Q_t from both sides of the defining recursion. □

3.3 Capacity and boundedness

Theorem 3.3 *(Uniform upper bound).* Suppose $\delta \in (0, 1]$. Then every admissible surplus-driven storage process satisfies

$$Q_t \leq (1-\delta)^t Q_0 + \eta \ln N \, \frac{1-(1-\delta)^t}{\delta}, \qquad t \geq 0.$$

In particular,

$$Q_t \le \max\left\{Q_0, \frac{\eta \ln N}{\delta}\right\} \qquad \text{for all } t.$$

If $\delta = 0$, then

$$Q_t \le Q_0 + t\eta \ln N.$$

Proof. Since $0 \le U_t \le 1$, we have $0 \le S_t \le \ln N$. Since $C_t \ge 0$, the recursion gives

$$Q_{t+1} \le (1-\delta)Q_t + \eta \ln N.$$

Iterating this inequality yields the stated bound when $\delta > 0$. The bound by the maximum follows because the right-hand side is a convex combination of Q_0 and $\eta \ln N/\delta$. When $\delta = 0$, the recurrence reduces to $Q_{t+1} \le Q_t + \eta \ln N$, which yields the linear bound by induction. □

Corollary 3.4 *(Sustainability ceiling).* Fix t and suppose $\eta > 0$. A nondecreasing step $Q_{t+1} \ge Q_t$ is achievable only if

$$C_t \le \eta \ln N - \delta Q_t.$$

Equivalently, if

$$C_t > \eta \ln N - \delta Q_t,$$

then no choice of interaction geometry at time t can prevent order from decreasing.

Proof. By Corollary 3.2, the condition $Q_{t+1} \ge Q_t$ requires $\eta S_t \ge \delta Q_t + C_t$. Since always $S_t \le \ln N$, this is possible only if $\eta \ln N \ge \delta Q_t + C_t$. Rearranging gives the stated criterion. □

3.4 Constant-input regime

Theorem 3.5 *(Steady state under constant forcing).* Suppose $\delta \in (0, 1]$, $U_t \equiv u_*$, and $C_t \equiv c_*$ are constant in time. Then

$$Q_t = (1-\delta)^t Q_0 + \frac{\eta F(u_*) - c_*}{\delta}\big(1 - (1-\delta)^t\big).$$

Consequently, if $\eta F(u_*) \ge c_*$, then Q_t converges to the nonnegative steady state

$$Q_\infty = \frac{\eta F(u_*) - c_*}{\delta}.$$

Proof. Apply Theorem 3.1 with constant $(\eta S_k - C_k) = \eta F(u_*) - c_*$. The geometric series identity gives the formula. If $\eta F(u_*) \ge c_*$, then the limit is nonnegative and the geometric factor tends to zero as $t \to \infty$. □

Remark 3.6. If $\eta F(u_*) < c_*$, then the affine recurrence has a negative equilibrium. That signals either that the regime is unsustainable or that a truncated nonnegative model must replace the linear one. The theorem does not hide this failure mode.

---

4. Exact Design Inequalities for Engineering Use

The previous section yields simple one-step control criteria.

Proposition 4.1 *(Target-increment inequality).* Fix t and a desired increment $r_t \geq 0$. Then

$$Q_{t+1} - Q_t \geq r_t$$

holds if and only if

$$\eta S_t \geq \delta Q_t + C_t + r_t.$$

If $\eta > 0$, this is equivalent to

$$S_t \geq \frac{\delta Q_t + C_t + r_t}{\eta}.$$

Since always $S_t \leq \ln N$, feasibility requires

$$\delta Q_t + C_t + r_t \leq \eta \ln N.$$

Proof. The first equivalence is Corollary 3.2 with r_t added to the right-hand side. The feasibility condition follows from the upper bound $S_t \leq \ln N$. □

Corollary 4.2 *(Maximum one-step achievable increment).* For any state (Q_t, C_t) with $\eta > 0$,

$$r_t^{\max} = \eta \ln N - \delta Q_t - C_t.$$

No policy based solely on Paper A's interaction geometry can guarantee a larger one-step increase.

Proof. Take the largest feasible r_t in Proposition 4.1 by setting the admissible upper limit $S_t = \ln N$. □

These identities provide an engineering operating-region specification: the model predicts exactly how much one-step order increase is attainable from the available geometric surplus budget.

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5. Testing Criteria and Preregistration Template

The dynamic framework is testable only after a target domain supplies explicit observables. The following conditions are required.

5.1 Required operational choices

A valid empirical test must preregister all of the following.

1. **State encoder.** A fixed map from raw system states to unit vectors and, where needed, to a block decomposition consistent with Paper A.
2. **Interaction observable.** A rule for computing $U_t \in [0,1]$ and hence $S_t = F(U_t)$ from encoded states.
3. **Order observable.** A domain-specific measurable proxy Y_t for stored order Q_t.
4. **Time scale.** A fixed sampling interval at which (U_t, Y_t, C_t) are defined.
5. **Cost observable or cost model.** Either a direct measure of C_t or a preregistered parametric approximation.

Without these choices, the dynamic framework is not falsifiable.

5.2 Minimal estimation model

With an observed order proxy Y_t, the simplest testable regression form is

$$Y_{t+1} = aY_t + bF(U_t) - \widehat{C}_t + \varepsilon_t,$$

with constraints

$$0 \le a \le 1, \qquad b \ge 0,$$

and the identification

$$a = 1 - \delta, \qquad b = \eta.$$

A serious test should fit this model on a training set, freeze all choices, and report held-out predictive performance.

5.3 Baselines and falsification criteria

At minimum, the surplus-driven dynamic model should be compared against:

1. **Persistence baseline:** $Y_{t+1} = Y_t$.
2. **Direct-input baseline:** $Y_{t+1} = \alpha + \beta U_t$.
3. **Matched autoregressive baseline:** $Y_{t+1} = \alpha + \beta Y_t$.
4. **Shuffled-input control:** replace U_t by a time-shuffled sequence with the same marginal distribution.

A preregistration-ready falsification rule is:

- the model fails if it does not outperform the matched autoregressive baseline on held-out prediction, or
- the fitted coefficient on $F(U_t)$ collapses to zero or negative under stability constraints, or
- the predicted sign condition from Corollary 3.2 fails systematically on held-out data.

This is a testing framework, not a proof of success.

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6. Minimal Engineering Specification

Any engineering use of the framework should document the following modules.

6.1 Representation module

- choice of state variables,
- normalization rule,
- block decomposition,
- evidence that the induced similarity measure is meaningful for the application.

6.2 Surplus-estimation module

- computation of U_t,
- computation of $S_t = \ln(1 + (N-1)U_t)$,
- numerical checks that $0 \le U_t \le 1$.

6.3 Storage-estimation module

- definition of the stored-order proxy Y_t,
- justification that Y_t is not merely a repackaging of U_t,
- calibration procedure for scale and noise.

6.4 Budget-control module

For a target floor $Q_{\min}$, the controller should monitor the one-step headroom

$$h_t := \eta \ln N - \delta Q_t - C_t.$$

- If $h_t < 0$, the current regime is unsustainable within this model.
- If $h_t = 0$, at best the controller can maintain the current state.
- If $h_t > 0$, any requested increment must satisfy $r_t \le h_t$.

These are exact consequences of Proposition 4.1 and Corollary 4.2.

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7. Limitations and Open Problems

1. The storage variable Q_t is not derived from first principles; it is a modeling state chosen for a target domain.
2. The linear update rule is the minimal tractable open-system law, not a universal law of nature.
3. The canonical concentration functional $\mathcal{N}_{\text{can}}$ is exact, but it measures concentration relative to a particular reference family, not a general physical entropy.
4. Multiscale, continuous-time, stochastic-noise, and multi-agent extensions remain open.
5. No result here implies cosmological, thermodynamic, or metaphysical claims.

The rigorous conclusion is therefore modest: one can define exact open-system surplus budgets and derive sharp operating inequalities, but any substantive application depends on explicit domain observables and must be tested.

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This paper provides a minimal dynamics framework and operating inequalities. Any stronger claim requires new assumptions and independent empirical confirmation.